Piedmont Classical High School

Summer Math Packet

Summer math packets are designed to brush up your math skills and prepare you to start the year with a firm foundation of appropriate knowledge and understanding. Summer math packets are optional and designed to contain material that should be review. If you don't understand or have forgotten something in the math packet, it is important that you get help and learn the concept so that you will be successful in your new math class. If you complete the packet, you will benefit from stronger skills, better understanding, <u>and a 100% quiz grade</u>.

Guidelines:

- You should complete the math packet for the class that you will be going into
- Math packets should reflect your own work
- You may get help with the material and concepts, but you must list people and resources who helped you on the accompanying form
- Completed Packets are due on August 19 at 4:00 pm and are to be turned into your math teacher.
- If you need help, you may use the following recommended resources or find some on your own
    - Last year's textbook (you may check one out if you don't have one)
    - Khan academy
    - Tutoring hours at the school (confirm times and dates on website)
        - August 1, 3 from 12:00- 2:00
        - August 8, 10,15 3:00-5:00
        - August 17,18 10:00-12:00
        - Other times by appointment

I will honor, through my words and actions, my school, my family, my country, and myself.

This packet reflects my own work and upholds the honor code.

Signed: ____

I received help from the following people:

____

____

STATISTICS

SUMMER MATH REVIEW

Welcome to Statistics! Here is your optional summer review packet. Be sure to show all work on a separate sheet of paper, read directions carefully, and you may use a calculator. You will need graph paper. Get help if you need it!

Statistics is not like other maths, so some parts of this packet only require thinking. The most important thing to take away from this packet is that COINCIIDENCE DOES NOT PROVE CAUSATION! This warning is often overlooked, but is crucial to the study and practice of statistics. (Example: The rooster crows just before sunrise; he does not cause the sun to rise.)

Basic Algebra Review

I. Order of Operations: PEMDAS

A. P (parentheses) - Simplify within parentheses or other grouping symbols first.

B. E (exponents) - Simplify all exponential expressions, following rules for exponents.

C. MD (multiply and divide) - Perform these operations as they occur, left to right.

D. AS (add and subtract) - Perform these operations as they occur, left to right.

Examples: $10 + 4 \div 2 - 3(2 \times 6 - 3)^2$

P $10 + 4 \div 2 - 3\ (12 - 3)^2$

P $10 + 4 \div 2 - 3(9)^2 = 10 + 4 \div 2 - 3 \times 81$

MD $10 + 2 - 243$

AS $12 - 243 = -231$

E. Practice

1. $2 + (2^2 + 6) \div 2 - 1$
2. $[16 - 4(3 + 2)] \div (-2)$
3. $(2 + 2^2 + 6) \div 2 - 1$
4. $3 \times 4^2 - (5 \times 1 - 4)$

II. Solving Equations

A. The goal is to get the variable (or the chosen variable) ALONE on one side of the equal marks.

B. Use mathematical operations to rearrange the terms - do NOT even think "move it over!" Add, subtract, multiply, or divide terms and coefficients as needed.

C. Whatever you do to one side of the equation (sides are separated by the =), you must do to the other.

Ex: 25 = 7 + x — Subtract 7 from both sides

-7 -7

18 = x

Ex: 5(x + 3) = 1/2 — Multiply both sides by 2 to eliminate the fraction

10(x + 3) = 1 — Distribute

10x + 30 = 1 — Subtract 30

10x = -29 — Divide by 10

x = -2.9 — Also correct would be $\frac{-29}{10}$

,

D. Practice

1. m - 16 = 5
2. -g + 21 = 13
3. -5 + 3k = 10
4. (x + 5)/6 = 3

III. Tackling word problems

A. Turn word problems into number problems as soon as possible.

B. Follow the question word by word, following instructions.

C. If the problem is a "story", write down what you know, *then* decide what to do.

D. Always write an equation. Simple problems that you can work in your head can give you safe practice writing the equations.

Ex: One half of what number is 6? 1/2 X = 6

What is 3 times the sum of 6 and 3? X = 3 (6 + 3)

Hamburgers cost $4.50 each. You have $25. How many hamburgers can you buy?

25 = 4.50 X In this case, your calculated answer, 5.555... seems to make no sense; nobody sells 5/9 of a hamburger. You can only buy 5 hamburgers and get 2.50 in change. Your correct final answer is 5; math always makes sense.

E. Practice

1. Twelve divided by what number is 4?

2. What number added to 16 makes 25?

3. What is 5 less than 38?

4. Joe had 14 M&M's. He gave 6 to Sally. How many does he have left?

IV. Linear Equations (equations whose solutions form a straight line)

A. Forms of linear equations

1. Standard form: Ax + By = C

a. A, B, and C are integers (no fractions!)

b. A (or B if there is no A, i.e., if A is zero) must be positive

2. Slope - intercept form: y = mx + b

a. Any linear equation can be put in this form by solving for y.

b. m is the slope of the line; b is the y-intercept (line crosses y axis)

c. If m is 0, then y = b and the line is horizontal (↔)

d. If m is undefined (i.e., y = 0), the x = a constant and the line is ↕ .

e. Fractions are allowed.

3. Point-slope form: $(y - y_1) = m (x - x_1)$

a. y_1 and x_1 are points on the line, m is the slope.

b. This can be turned into slope-intercept form easily.

c. Fractions are allowed

4. Slope formula: $\frac{y_2 - y_1}{x_2 - x_1}$, which gives you "rise over run"

.

B. Graphing

1. Plotting points

a. Choose a small number for x and plug it into equation, repeat.

b. Graph points, then connect dots.

2. Using point-slope form:

a. Find y-intercept, then

b. Move to other points according to slope.

c. Connect dots.

C. Practice (on graph paper)

1. Graph: $y = 3x + 2$

2. Graph: $3x + 3y = 12$

3. Graph: $y = 4$ and $x = 3$ on the same set of axes

4. Write an equation (in any form) for the line with slop -2 that passes through the point (1, 3).

5. Write an equation for a vertical line passing through (-4, 5)

V. Literal Equations (solving for a variable instead of a number)

A. Follow general rules for solving any equation.

B. Remember, anything you can do with numbers you can do with variable.

Ex. D = RT solve for R

Divide both sides by T

D/T = R

Ex: $A = \pi r^2$ solve for r

Divide both sides by π

$r^2 = A/\pi$

Take the square root of both sides

$r = \sqrt{A/\pi}$

C. Practice:

1. 5x + 7y = 35 solve for y
2. 5x - 7y = w solve for x

VI. Exponents

A. An exponent tells how many time a term is used as a factor.

3^3 means 3 X 3 X 3 = 27

B. Exponential expressions can be added, subtracted, multiplied, divided, squared, and "square rooted."

C. Whatever you do to the exponential expression, do ONE STEP LOWER to the exponents.

Ex: $(x^2)(x^3) = x^{2+3} = x^5$

Ex: $(x^2)^3 = x^6$

Ex: $x^2 + x^2 = 2x^2$ (The exponent did not change.)

Ex: $x^2 \div x^3 = x^{2-3} = x^{-1} = 1/x$

D. Anything (and that means ANYTHING) raised to the 0 power = 1

E. Practice - simplify

1. $x^2 \cdot x^7$
2. $b^5 + 2b^2 + 3b^5$
3. $c^4 \times c^3$
4. $g^2(g^3 + 2g^2 - 3g + 4)$

VII. Squares and square roots.

A. Learn the perfect squares:

1. They are, from 1^2 to 15^2, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225
2. Learn means memorize

B. To find square roots,

1. Factor any perfect squares out of the number under the radical
2. Take those squares, then leave the rest under.

Ex: $\sqrt{98x^3} = \sqrt{(49)(2)(x)(x)(x)} = 7x\sqrt{2x}$

C. Practice

1. the square root of 32
2. the square root of 450
3. the square root of 48

VIII. Polynomials

A. Multiplying: FOIL binomials

Ex: $(x+3)(x-2) = x^2 - 2x + 3x - 6 = x^2 + x - 6$

$(a + b)(c + d) = ac + ad + bc + bd$

B. Factoring: Breaking a polynomial into its factors

1. Remove the GCF, if there is one

2. Look for a pattern

   a. Difference of Two Squares (DOTS): $a^2 - b^2 = (a+b)(a-b)$

   b. Perfect Trinomial Square (PTS): $a^2+2ab + b^2 = (a+b)(a+b) = (a+b)^2$

   or $a^2-2ab + b^2 = (a-b)(a-b) = (a-b)^2$

   Ex: $x^2 - x - 6 = (x-3)(x+2)$

   Ex: $3x^3 - 3x^2 - 18x = 3x(x^2 - x - 6) = 3x\ (x-3)(x+2)$

   Ex: $x^2 - 81 = (x+9)(x-9)$ (DOTS)

   Ex: $x^2 + 6x + 9 = (x+3)(x+3) = (x+3)^2$

3. Practice: Factor completely

   1. $4x^2 - 16$ 2. $x^2 - 8x + 16$

   3. $5x^2y - xy^2 + 3x^2y^2$ 4. $b^2 -38b + 72$

IX; . Logarithms: The logarithmic function is the inverse of the power function.

Ex: $10^2= 100$, therefore $\log(100) = 2$

A. Logarithms can be to any base, but if the base is not specified, it is 10.

   Ex: $2^3 = 8$, therefore $\log_2(8) = 3$

B. Logarithms to base *e* are known as natural logs, and are designated ln.

   Ex: $e^0 = 1$, therefore, $\ln(1) = 0$

C. Rules of logarithms: See page 7 for that and other helpers.

D. Practice:

   1. $\log_3(81) =$ 2. $\ln(1) =$ 3. $\log(1000) =$

Things to Think About

1. If a meteor plunges to earth, is it more likely to land on land or in water?

2. If the meteor crashes into dry land, is it more likely to be in Asia or Australia?

3. If it lands in the continental United States, is it more likely to be in Texas or Ohio?

4. Probability

    A. Probability is the chance of something happening

        1. If you flip a fair coin, the probability of heads is .5. So is the probability of tails. The total of all probabilities is 1. (No, don't start with "what if it stands on edge?"; it won't.)

    B. If a die (singular of dice) is rolled, the probability of a 4 showing is 1/6. There are six sides, each with spots from one to six.

    C. Practice and think (or play with dice or make charts):

        What is the probability that rolling two dice will give a total of 8?

EXPONENTS AND RADICALS

$a^m a^n = a^{m+n}$ $\qquad a^{1/n} = \sqrt[n]{a}$

$(a^m)^n = a^{mn}$ $\qquad a^{m/n} = \sqrt[n]{a^m}$

$(ab)^n = a^n b^n$ $\qquad a^{m/n} = (\sqrt[n]{a})^m$

$\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$ $\qquad \sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$

$\dfrac{a^m}{a^n} = a^{m-n}$ $\qquad \sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$

$a^{-n} = \dfrac{1}{a^n}$ $\qquad \sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

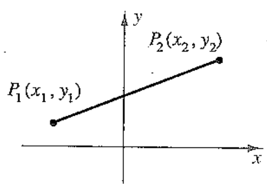
QUADRATIC FORMULA

If $a \neq 0$, the roots of $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

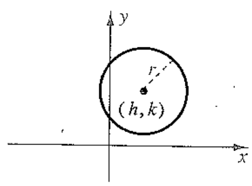
DISTANCE FORMULA

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



CIRCLE

$$(x - h)^2 + (y - k)^2 = r^2$$



INEQUALITIES

If $a > b$ and $b > c$, then $a > c$.

If $a > b$, then $a + c > b + c$.

If $a > b$ and $c > 0$, then $ac > bc$.

If $a > b$ and $c < 0$, then $ac < bc$.

SPECIAL PRODUCT AND FACTORING FORMULAS

$(x + y)(x - y) = x^2 - y^2$

$(x + y)^2 = x^2 + 2xy + y^2$

$(x - y)^2 = x^2 - 2xy + y^2$

$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

$x^2 - y^2 = (x + y)(x - y)$

$x^2 + 2xy + y^2 = (x + y)^2$

$x^2 - 2xy + y^2 = (x - y)^2$

$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

ABSOLUTE VALUE $(d > 0)$

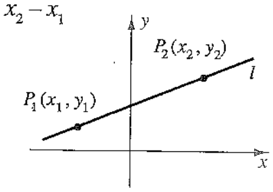
$|x| < d$ if and only if $-d < x < d$.

$|x| > d$ if and only if either

$x > d$ or $x < -d$.

SLOPE m OF A LINE

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



LOGARITHMS

$y = \log_a x$ means $a^y = x$.

$\log_a xy = \log_a x + \log_a y$

$\log_a \dfrac{x}{y} = \log_a x - \log_a y$

$\log_a x^r = r \log_a x$

$a^{\log_a x} = x$

$\log_a a^x = x$

$\log_a 1 = 0$

$\log_a a = 1$

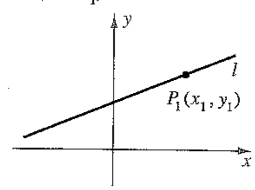
$\log x = \log_{10} x$

$\ln x = \log_e x$

$\log_b u = \dfrac{\log_a u}{\log_a b}$

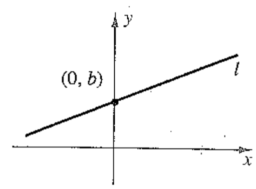
POINT-SLOPE FORM OF A LINE

$$y - y_1 = m(x - x_1)$$



SLOPE-INTERCEPT FORM OF A LINE

$$y = mx + b$$



General Reference Information