Piedmont Classical High School
Summer Math Packet

Summer math packets are designed to brush up your math skills and prepare you to start the year with a firm foundation of appropriate knowledge and understanding. Summer math packets are optional and designed to contain material that should be review. If you don't understand or have forgotten something in the math packet, it is important that you get help and learn the concept so that you will be successful in your new math class. If you complete the packet, you will benefit from stronger skills, better understanding, <u>and a 100% quiz grade</u>.

Guidelines:

- You should complete the math packet for the class that you will be going into
- Math packets should reflect your own work
- You may get help with the material and concepts, but you must list people and resources who helped you on the accompanying form
- Completed Packets are due on August 19 at 4:00 pm and are to be turned into your math teacher.
- If you need help, you may use the following recommended resources or find some on your own
    - Last year's textbook (you may check one out if you don't have one)
    - Khan academy
    - Tutoring hours at the school (confirm times and dates on website)
        - August 1, 3 from 12:00- 2:00
        - August 8, 10,15 3:00-5:00
        - August 17,18 10:00-12:00
        - Other times by appointment

I will honor, through my words and actions, my school, my family, my country, and myself.

This packet reflects my own work and upholds the honor code.

Signed: ____

I received help from the following people:

____

____

Name ______________________________

Algebra II Summer Math Review

Show all work neatly and thoroughly using a separate sheet of paper. Attach your separate sheets of paper to this packet. Circle/box your final answers. These problems should be completed without calculators.

I. Vocabulary

Please review the following terms.

1. Natural numbers- positive integers or "counting numbers." 1, 2, 3, 4, 5, …
2. Whole numbers- zero plus natural numbers: 0, 1, 2, 3, 4, 5, …
3. Integers- negative and positive whole numbers: -4, -3, -2, -1, 0, 1, 2, 3, etc.
4. Rational numbers- a number that can be expressed as a fraction
5. Irrational numbers- numbers that cannot be expressed as terminating or repeating decimals. In other words, irrational numbers continue on forever without fitting a pattern.
6. Absolute value- the absolute value of a number is its distance from zero. Ex. $|-4| = 4$ and $|4| = 4$
7. What is the difference between an expression and an equation? An expression does not have an equal sign and an equation does.
8. Polynomial- a monomial (a number, variable or product of one or more numbers and variables) or monomials added together.
9. Slope- rate of change, $\frac{rise}{run}$, how steep a line is. Slope can be positive, negative, zero, or undefined. It's represented by the variable "m."
10. Parallel lines- lines that will never cross, lines that have the same slope.
11. Perpendicular lines- lines that intersect at right (90 degree) angles. Their slopes are opposite reciprocals.
12. Relation- a set of ordered pairs.
13. Function- a relation where every domain is paired with exactly one range (every x corresponds to exactly one y)
14. Domain- all of the x's in a relation
15. Range- all of the y's in a relation. Range can also mean the difference between the highest and lowest data value within a set of data.
16. What is the difference between an equation and an inequality? An equation contains an equal sign. Both sides of the equation are equal to each other. An inequality uses the following symbols: $<, >, \leq, \geq$

II. Order of Operations

"PEMDAS"

- Parentheses: Simplify within each grouping symbol working from the inside out.
- Exponents: Simplify using exponent rules.
- Multiplication and Division (from left to right).
- Addition and Subtraction (from left to right).

Examples:

Ex. 1. Simplify. $2 * (2-3)^3 + 6 \div 3 + (8-5)^2$

$$2 * (-1)^3 + 6 \div 3 + (3)^2$$

$$2 * -1 + 6 \div 3 + 9$$

$$-2 + 2 + 9$$

$$\boxed{9}$$

Ex. 2. Simplify. $\frac{(9-7)^2}{8} + |36 - 18 * 8|$

$$\frac{2^2}{8} + |36 - 144|$$

$$\frac{4}{8} + |-108|$$

$$\frac{1}{2} + 108$$

$$\frac{1}{2} + \frac{216}{2}$$

$$\boxed{\frac{217}{2}}$$

Practice:

Simplify. Do not use a calculator.

1. $|-11| - |-29|$

2. $2[4 + (2^3 - 1)]$

3. $50 - (1 + 9)$

4. $3 + 2 * 3 + 5$

5. Evaluate if x=4 and y=3.

$$x^2y - xy^2$$

III. Solving Linear Equations

- To solve an equation means to find the value of the variable that makes the equation "true" when that value is plugged back in. Isolate the variable by performing opposite operations. Remember that whatever you do to one side of the equation you must do to the other.
  - Always distribute and combine like terms first, if necessary.
  - If there is more than one variable term, move the variable terms to the same side of the equal sign using opposite operations. Get the constants ("plain numbers") together on the other side of the equal sign.
  - Perform opposite operations to each side in order to isolate and solve for the variable.

Examples:

Solve. $5(2z - 9) = 7z - 60$

$10x - 45 = 7z - 60$ (Distribute)

$3z - 45 = -60$ (Move the variable terms to one side by opposite operations- subtract 7z.)

$3z = -15$ (Begin to isolate the variable by adding 45 to both sides.)

$\boxed{z = -5}$ (Divide both sides by 3.)

Solve. $\frac{3y-2(y-1)}{6} = -1$

$3y - 2(y - 1) = -6$ (Multiply both sides by the denominator 6 to cancel the fraction.)

$3y - 2y + 2 = -6$ (Distribute -2.)

$y + 2 = -6$ (Combine like terms.)

$\boxed{y = -8}$ (Subtract 2 from both sides to isolate the variable.)

Practice:

Solve.

1. $2(m + 3) + 1 = 23$

2. $2t - 8 = 0$

3. $142 + d = 97$

4. $t - 4 = -7$

5. $3(t - 1) = -(t - 5)$

IV. Writing and Graphing Linear Equations

- The slope of a linear equation is described by either the line's $\frac{rise}{run}$ or by the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$. Slope is the rate of change between two quantities.
- Linear equations are often represented in one of three forms:
  - Slope intercept form: $y = mx + b$ where m is the slope and b is the y-intercept (where the line crosses the y-axis).
  - Point-slope form: $y - y_1 = m(x - x_1)$ where m is the slope and (x_1, y_1) is a given point.
  - Standard form: $Ax + By = C$, where A and B can't both be 0, there cannot be any fractions, and A, B, and C should not have a greatest common factor.
- To graph a linear equation, first plot the y-intercept. From that point on the coordinate plane, use the slope to rise then run to the next point. Sketch the line.
- Parallel lines have the same slope, and perpendicular lines' slopes are opposite reciprocals.

Example:

Write the equation of a line that passes through points (5, 0) and (4, 5).

First find the slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \frac{5-0}{4-5}$ → $m = \frac{5}{-1}$ → $m = -5$

Choose one of the two points to plug into $y = mx + b$. (You can also write an equation with slope and one point using the point-slope form.) I will choose the first point (5, 0).

$0 = -5(5) + b$ (Substitute the ordered pair and slope in for the x, y, and m variables.

$0 = -25 + b$ (PEMDAS)

$25 = b$ (Isolate the variable.)

$\boxed{y = -5x + 25}$ (Substitute the slope and y intercept into the slope intercept form.

Practice:

Find the slope of the line that passes through the two given points.

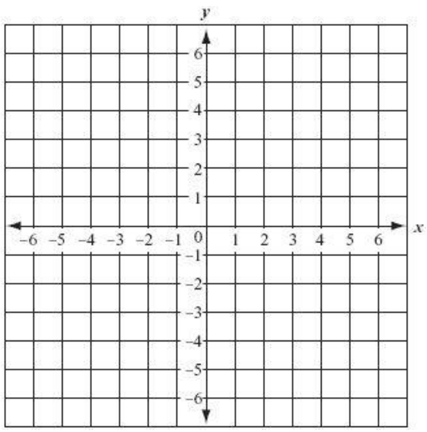
1. (1, 3), (-2, -6) 2. (-6, 4), (-6, -2) 3. (8, -3), (-2, -3)

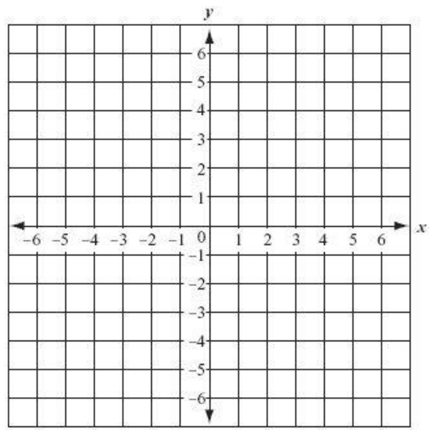
Graph each equation.

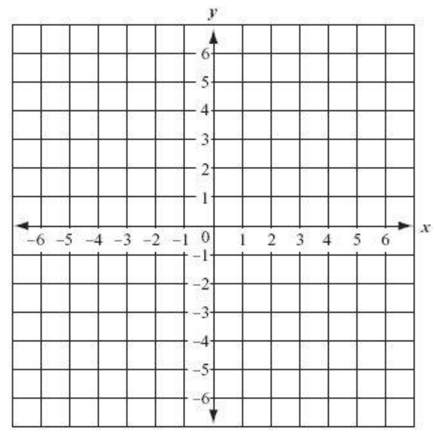
4. $y = 2x$

5. $y = -\frac{4}{5}x + 5$

6. $3x + y = 3$ (Solve for y 1st!)







Write the equation of a line with the given conditions.

7. The slope is 3 and the y-intercept is 2.

8. The line passes through (-3. 3) with a slope of 1.

9. The line passes through the points (-4, 2) and (1, 12).

10. The line passes through (5, -2) and is parallel to $y = 2x + 7$.

11. The line passes through (0, -3) and is perpendicular to $y = -2x - 7$.

V. Properties of Exponents

$x^m * x^n = x^{m+n}$ When multiplying the same base ADD their exponents.

$\frac{x^m}{x^n} = x^{m-n}$ When dividing the same base SUBTRACT their exponents.

$(x^m)^n$ When distributing one exponent to another MULTIPLY them.

$x^{-m} = \frac{1}{x^m}$ A simplified answer cannot contain negative exponents. "Flip" the term with the negative exponent either over or under the fraction bar to make it positive.

$x^0 = 1$ Anything raised to the zero power is one.

***Simplified monomial expressions will contain only one of each variable, will not have any negative exponents, will not have any parentheses, and all constant fractions will be reduced.

Examples:

Simplify.

Ex. 1. $t^4 * t^3 * t = \boxed{t^8}$

Ex. 2. $(2ab^2)(3a^2b^3) = \boxed{6a^3b^5}$

Ex. 3. $\frac{(3y)^0}{6a} = \boxed{\frac{1}{6a}}$

Practice:

1. $c^5 * c^4$
2. $\left(\frac{a^2}{a}\right)^2$
3. $\frac{-6n^5x^3}{18nx^7}$

VI. Polynomials

A polynomial is an expression that has variables, coefficients, and exponents. You can add and subtract them by combining like terms. Multiply them by the FOIL (First Outer Inner Last) method.

Examples:

Ex. 1. Simplify $(2x^2 - 3x + 5) + (-8x - 7x^2 - 23)$

$\boxed{-5x^2 - 11x - 18}$

Ex. 2. $(3x + 7)(4x - 2)$

$12x^2 - 6x + 28x - 14$

$\boxed{12x^2 - 22x - 14}$

Practice:

1. Simplify. $(2x^2 - 5x + 7) - (3x^3 + x^2 + 2)$
2. Multiply. $(3x + 2)(x - 2)$
3. Multiply. $(2c + 9)(2c - 9)$

VII. Factoring Polynomials

- Factoring polynomials "breaks downs" the polynomial into its simplest set of factors. It's essentially the opposite of FOILing or multiplying polynomials.
- Always look for a GCF (greatest common factor) first. If there is one, factor it out (but don't "drop it" and continue, if possible.

Examples:

Ex. 1. Factor completely. $x^2 + 7x + 6$. Ask yourself what two numbers multiply to get 6 and add to get 7? 6 and 1.

$(x + 6)(x + 1)$

Ex. 2. $100x^2 - 9$. This is a special case. It's the difference of two squares.

$(10x - 3)(10x + 3)$

Factor completely.

1. 13x+26y
2. $x^2 + 7y + 12$
3. $x^2 + 5x - 6$
4. $2h^2 + 13h - 24$
5. $49f^2 - 36$

VIII. Radicals

$$1^2 = 1$$

$$2^2 = 4$$

$$3^2 = 9$$

$$12^2 = 144$$

You should be able to recognize the perfect squares (one number multiplied by itself) from 1-15.

Simplifying a radical requires you to "break down" or factor a number down as far as it can go until you reach prime factors (prime numbers cannot be divided by anything other than one and the number itself.) To simplify a square root take out the amount of pairs possible of numbers or variables.

Examples:

Ex. 1. $\sqrt{81} = 9$

Ex. 2. $\sqrt{12} = \sqrt{2 * 2 * 3} = 2\sqrt{3}$

Practice:

1. $\sqrt{64}$
2. $\sqrt{121}$
3. $\sqrt{45a^3}$
4. $\sqrt{72x^2y^5}$