AP CALCULUS

SUMMER MATH REVIEW

Welcome to AP Calculus! Here is your optional summer review packet. Be sure to show all work on a separate sheet of paper, read directions carefully, and do NOT use a calculator unless such use is specified. You will need graph paper. Get help if you need it!

When you should (or need to) use a calculator:

1. When you need numerical values for trigonometric or logarithmic functions
2. When you need to check your work on graphing.
3. When your workload is such that the extra speed is important.

You do need at least a scientific calculator, although a graphing calculator is highly recommended. The more recent, and more expensive, ones have fun bells and whistles, but most any will suffice. TI models will most closely follow classroom instruction.

Whatever calculator you have, you need to be adept on it. Practice for fun, read the instruction manual, or just follow instructions in class.

I. Domain of functions:

A. The domain of a function is the set of X values that will make the function real.

1. Any number that would make a denominator equal 0 is not in the domain, since division by zero is undefined.
2. Any number that would produce an imaginary number in a domain is not in the domain, since imaginary numbers are by definition not real.

Ex: $1 \div (x+3)$ domain: $x \neq -3$ (There are several ways to write this)

$1 \div (x+3)^2$domain: $x \geq -3$

B. Practice - Give the domain for each.

1. f(x) = $\frac{3+x^2}{x^2-16}$
2. f(x) = $5x^3-x^2+x+32$
3. f(x) = $\frac{9}{\sqrt{16-x^2}}$
4. f(x) = $\frac{x-5}{\sqrt[3]{x-7}}$

II. Graphing (See Parent Graphs, page 6 in this packet.)

A. Linear equations: Form a straight line, have at most 1 solution if = 0

1. Slope-intercept form: $y = mx + b$ (m is slope, b is y-intercept.)

2. Point-slope form: $(y-y_1) = m(x-x_1)$ (use if at least one point is given and slope is either given or can be calculated)

3. Standard form: $Ax + By = C$ (no fractions, leading coefficient >0)

B. Quadratic equations: Form a parabola, have 1, 2, 0 solutions.

1. To solve by factoring: Factor, then set each factor equal to zero.

2. To solve by completing the square: Complete square, solve for y.

3. To solve by quadratic formula: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

4. Standard form: $ax^2 + bx + c = 0$

5. Vertex form: $y = a(x-h)^2 + k$

C. Cubic equations: Have 1, 2, or 3 solutions

1. End behavior: Increasing from left to right if leading coefficient > 0;

   decreasing from left to right if leading coefficient < 0.

2. May be odd, i.e., symmetrical about the origin; may never be even.

D. Odd and Even functions

1. In an even function, $f(-x) = f(x)$, symmetrical about y-axis

   Ex: $y = x^2$

2. In an odd function, $f(-x) = -f(x)$, symmetrical about origin.

3. A function lacking these symmetries are neither odd nor even.

Practice: On graph paper, graph each equation and tell if it's odd or even, then solve it.

1. $y = x^2 + 4$ 2. $y = x^3 - 4$ 3. $y = (x+2)^2 + 2$

III. Functions: Anything you can do with numbers in arithmetic, you can do with variables in algebra and with functions in calculus!

A. Adding and subtracting functions: Add or subtract normally.

Ex: $f(x) = x + 4$ $g(x) = x^2 + x - 3$

$f(x) + g(x) = x+4 + x^2 + x - 3 = x^2 + 2x + 1$

B. Multiplying and dividing: Replace variable in one function with the other function.

Ex: $f(g(x)) = f(x^2 + 2x - 3) = (x^2 + x - 3) + 4 = x^2 + x + 1$

C. Practice - let $f(x) = \sqrt{x}$ and $h(x) = x^2 + 6x + 9$

1. Find f(x) - g(x) 2. Find f(g(x))

IV. Exponents:

A. An exponent tells how many time a term is used as a factor.

3^3 means 3 X 3 X 3 = 27

B. Exponential expressions can be added, subtracted, multiplied, divided, squared, and "square rooted."

C. Whatever you do to the exponential expression, do ONE STEP LOWER to the exponents.

Ex: $(x^2)(x^3) = x^{2+3} = x^5$

Ex: $(x^2)^3 = x^6$

Ex: $x^2 + x^2 = 2x^2$ (The exponent did not change.)

Ex: $x^2 \div x^3 = x^{2-3} = x^{-1} = 1/x$

D. Anything (and that means ANYTHING) raised to the 0 power = 1

E. Fractional exponents represent roots

Ex: $x^{\frac{3}{4}} = \sqrt[4]{x^3}$

F. Practice: Simplify

1. $x^2 \cdot x^7$ 2. $b^5 + 2b^2 + 3b^5$

3. $(b^{\frac{1}{4}})(b^{\frac{2}{3}})$ 4. $(g^3)^3$

V. Trigonometry (See Unit Circle, page 5)

A. Know SOH, CAH, TOA

B. On the unit circle (see next page), the sinθ = y, cosθ = x, and tanθ = $\frac{y}{x}$

C. Reciprocals: 1/ sinθ = cscθ, 1/cosθ = secθ , and 1/tanθ = cotθ

D. Practice - find the following:

1. $\sin 30^o$ 2. $\tan 45^o$ 3. $\cos\pi$

VI. Graphing trig. functions - calculator active

A. Use your calculator to graph the following, then use words to describe the graph.

1. y = sin(x) 2. y = sin(x+4) 3. y = 3sin(x)

2. y = tan(x) 5. y = -2tan(x) 6. y = -cos(x)

VII. Logarithms: The logarithmic function the inverse of the power function.

Ex: $10^2 = 100$, therefore log(100) = 2

A. Logarithms can be to any base, but if the base is not specified, it is 10.

Ex: $2^3 = 8$, therefore $\log_2(8) = 3$

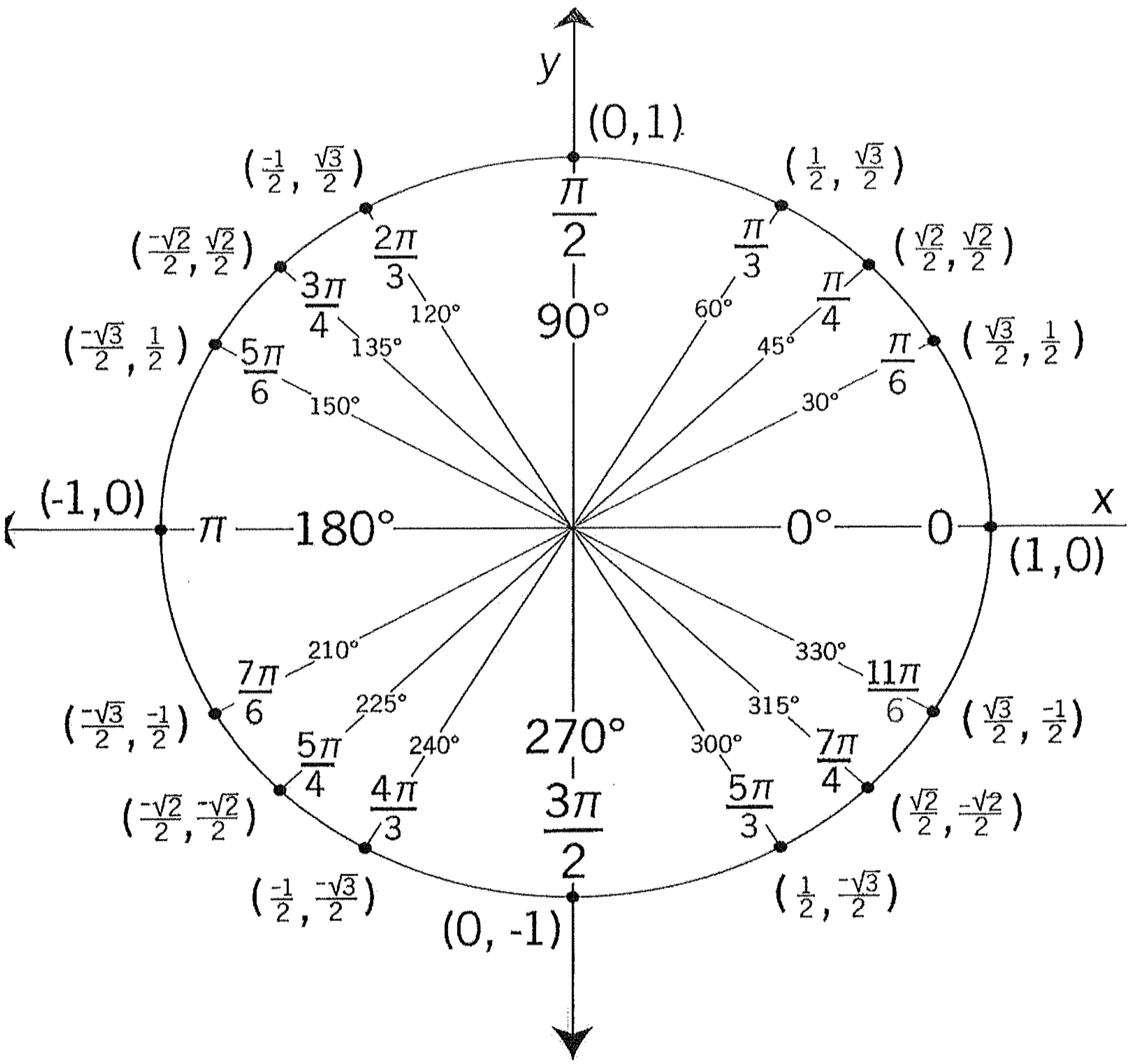
B. Logarithms to base *e* are known as natural logs, and are designated ln.

Ex: $e^0 = 1$, therefore, ln(1) = 0

C. Rules of logarithms: See page 7 for that and other helpers.

D. Practice:

1. $\log_3(81)$ 2. ln(1) = 3. log(1000) =


y
x
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(1,0)
(-1,0)
(0, -1)
0°
90°
180°
270°
0
π/2
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3π/2
30°
45°
60°
120°
135°
150°
210°
225°
240°
300°
315°
330°
π/6
π/4
π/3
2π/3
3π/4
5π/6
7π/6
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4π/3
5π/3
7π/4
11π/6
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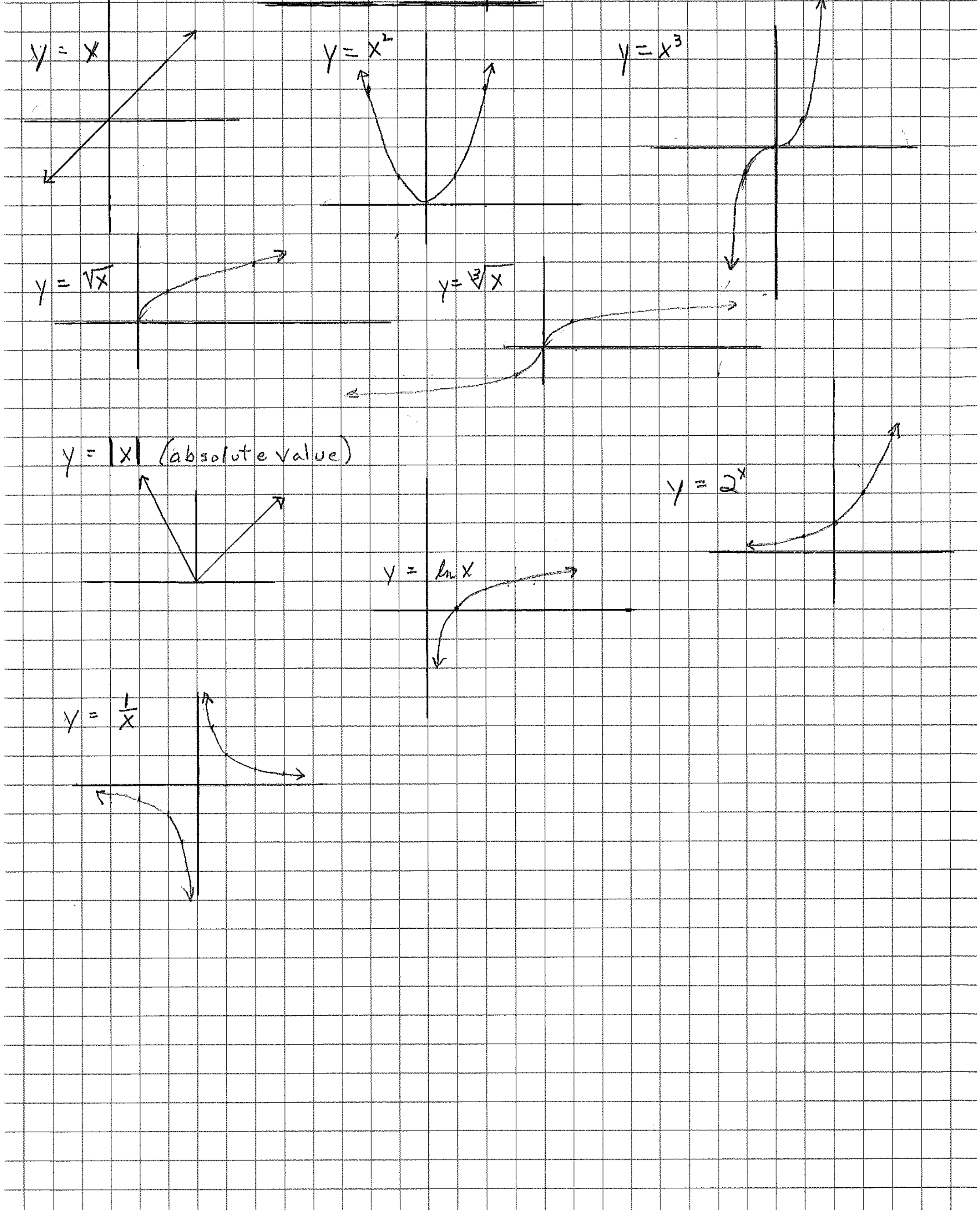
UNIT CIRCLE

FORM 1952

Parent Graphs



EXPONENTS AND RADICALS

$a^m a^n = a^{m+n}$ $\qquad a^{1/n} = \sqrt[n]{a}$

$(a^m)^n = a^{mn}$ $\qquad a^{m/n} = \sqrt[n]{a^m}$

$(ab)^n = a^n b^n$ $\qquad a^{m/n} = (\sqrt[n]{a})^m$

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ $\qquad \sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$

$\frac{a^m}{a^n} = a^{m-n}$ $\qquad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

$a^{-n} = \frac{1}{a^n}$ $\qquad \sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

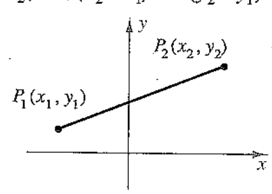
QUADRATIC FORMULA

If $a \neq 0$, the roots of $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

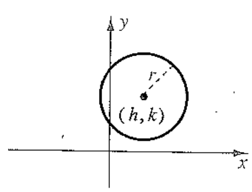
DISTANCE FORMULA

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



CIRCLE

$$(x-h)^2 + (y-k)^2 = r^2$$



INEQUALITIES

If $a > b$ and $b > c$, then $a > c$.

If $a > b$, then $a + c > b + c$.

If $a > b$ and $c > 0$, then $ac > bc$.

If $a > b$ and $c < 0$, then $ac < bc$.

SPECIAL PRODUCT AND FACTORING FORMULAS

$(x+y)(x-y) = x^2 - y^2$

$(x+y)^2 = x^2 + 2xy + y^2$

$(x-y)^2 = x^2 - 2xy + y^2$

$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

$x^2 - y^2 = (x+y)(x-y)$

$x^2 + 2xy + y^2 = (x+y)^2$

$x^2 - 2xy + y^2 = (x-y)^2$

$x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

$x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

ABSOLUTE VALUE $(d > 0)$

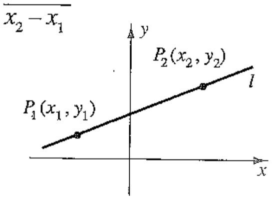
$|x| < d$ if and only if $-d < x < d$.

$|x| > d$ if and only if either

$x > d$ or $x < -d$.

SLOPE m OF A LINE

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



LOGARITHMS

$y = \log_a x$ means $a^y = x$.

$\log_a xy = \log_a x + \log_a y$

$\log_a \frac{x}{y} = \log_a x - \log_a y$

$\log_a x^r = r \log_a x$

$a^{\log_a x} = x$

$\log_a a^x = x$

$\log_a 1 = 0$

$\log_a a = 1$

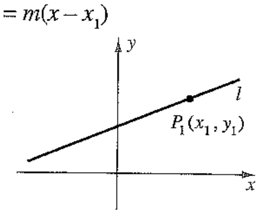
$\log x = \log_{10} x$

$\ln x = \log_e x$

$\log_b u = \frac{\log_a u}{\log_a b}$

POINT-SLOPE FORM OF A LINE

$$y - y_1 = m(x - x_1)$$



SLOPE-INTERCEPT FORM OF A LINE

$$y = mx + b$$