

GEOMETRY

SUMMER MATH REVIEW

Welcome to Geometry! Here is your optional summer review packet. Be sure to show all work on a separate sheet of paper, read directions carefully, and do NOT use a calculator. You will need graph paper. Get help if you need it!

I. Order of Operations: PEMDAS

- A. P (parentheses) - Simplify within parentheses or other grouping symbols first.
- B. E (exponents) - Simplify all exponential expressions, following rules for exponents.
- C. MD (multiply and divide) - Perform these operations as they occur, left to right.
- D. AS (add and subtract) - Perform these operations as they occur, left to right.

Examples: $10 + 4 \div 2 - 3(2 \times 6 - 3)^2$

P $10 + 4 \div 2 - 3(12 - 3)^2$

P $10 + 4 \div 2 - 3(9)^2 = 10 + 4 \div 2 - 3 \times 81$

MD $10 + 2 - 243$

AS $12 - 243 = -231$

E. Practice

1. $2 + (2^2 + 6) \div 2 - 1$

2. $[16 - 4(3 + 2)] \div (-2)$

3. $(2 + 2^2 + 6) \div 2 - 1$

4. $3 \times 4^2 - (5 \times 1 - 4)$

II. Solving Equations

- A. The goal is to get the variable (or the chosen variable) ALONE on one side of the equal marks.

- B. Use mathematical operations to rearrange the terms. Add, subtract, multiply, or divide terms and coefficients as needed.
- C. Whatever you do to one side of the equation (sides are separated by the =), you must do to the other.

Ex: $25 = 7 + x$ Subtract 7 from both sides

$$\underline{-7 \quad -7}$$

$$18 = x$$

Ex: $5(x + 3) = 1/2$ Multiply both sides by 2 to eliminate the fraction

$$10(x + 3) = 1 \quad \text{Distribute}$$

$$10x + 30 = 1 \quad \text{Subtract 30}$$

$$10x = -29 \quad \text{Divide by 10}$$

$$x = -2.9 \quad \text{Also correct would be } \frac{-29}{10}$$

D. Practice

1. $m - 16 = 5$
2. $-g + 21 = 13$
3. $-5 + 3k = 10$
4. $\frac{x+5}{6} = 3$

III. Tackling word problems

- A. Turn word problems into number problems as soon as possible.
- B. Follow the question word by word, following instructions.
- C. If the problem is a "story", write down what you know, *then* decide what to do.

D. Always write an equation. Simple problems that you can work in your head can give you safe practice writing the equations.

Ex: One half of what number is 6? $\frac{1}{2}X = 6$

What is 3 times the sum of 6 and 3? $X = 3(6 + 3)$

Hamburgers cost \$4.50 each. You have \$25. How many hamburgers can you buy?

$25 = 4.50 X$ In this case, your calculated answer, 5.555... seems to make no sense; nobody sells 5/9 of a hamburger. You can only buy 5 hamburgers and get 2.50 in change. Your correct final answer is 5; math always makes sense.

E. Practice

1. Twelve divided by what number is 4?
2. What number added to 16 makes 25?
3. What is 5 less than 38?
4. Joe had 14 M&M's. He gave 6 to Sally. How many does he have left?

IV. Linear Equations (equations whose solutions form a straight line)

A. Forms of linear equations

1. Standard form: $Ax + By = C$
 - a. A, B, and C are integers (no fractions!)
 - b. A (or B if there is no A, i.e., if A is zero) must be positive
2. Slope - intercept form: $y = mx + b$
 - a. Any linear equation can be put in this form by solving for y.
 - b. m is the slope of the line; b is the y-intercept (line crosses y axis)
 - c. If m is 0, then $y = b$ and the line is horizontal (\longleftrightarrow)

d. If m is undefined (i.e., no y term in equation), then x equals a constant and the line is vertical.



e. Fractions are allowed in this form.

3. Point-slope form: $(y - y_1) = m(x - x_1)$

a. y_1 and x_1 are points on the line, m is the slope.

b. This can be turned into slope-intercept form easily.

c. Fractions are allowed in this form.

4. Slope formula: $\frac{y_2 - y_1}{x_2 - x_1}$, which gives you "rise over run"

B. Graphing

1. Plotting points

a. Choose a small number for x and plug it into equation, repeat.

b. Graph points, then connect dots.

2. Using point-slope form:

a. Find y -intercept, then

b. Move to other points according to slope.

Example: If slope is $\frac{3}{2}$, start at intercept and go up 3, right 2.

c. Connect dots.

C. Practice (on graph paper)

1. Graph: $y = 3x + 2$

2. Graph: $3x + 3y = 12$

3. Graph: $y = 4$ and $x = 3$ on the same set of axes

4. Write an equation (in any form) for the line with slope -2 that passes through the point $(1, 3)$.

5. Write an equation for a vertical line passing through $(-4, 5)$

V. Literal Equations (solving for a variable instead of a number)

A. Follow general rules for solving any equation.

B. Remember, anything you can do with numbers you can do with variable.

Ex. $D = RT$ solve for R

Divide both sides by T

$$\frac{D}{T} = R$$

Ex: $A = \pi r^2$ solve for r

Divide both sides by π

$$r^2 = \frac{A}{\pi}$$

Take the square root of both sides

$$r = \sqrt{A/\pi}$$

C. **Practice:**

1. $5x + 7y = 35$ solve for y

2. $5x - 7y = w$ solve for x

VI. Exponents

A. An exponent tells how many times a term is used as a factor.

$$3^3 \text{ means } 3 \times 3 \times 3 = 27$$

B. Exponential expressions can be added, subtracted, multiplied, divided, squared, and "square rooted."

C. Whatever you do to the exponential expression, do ONE STEP SIMPLER to the exponents. When multiplying, add exponents, etc.

$$\text{Ex: } (x^2)(x^3) = x^{2+3} = x^5$$

$$\text{Ex: } (x^2)^3 = x^6$$

Ex: $x^2 + x^2 = 2x^2$ (The exponent did not change.)

$$\text{Ex: } x^2 \div x^3 = x^{2-3} = x^{-1} = \frac{1}{x}$$

Note: When dividing, subtract exponents. A negative exponent triggers a reciprocal.

D. Anything (and that means ANYTHING) raised to the 0 power = 1

E. Practice - simplify

1. $x^2 \cdot x^7$

2. $b^5 + 2b^2 + 3b^5$

3. $c^4 \times c^3$

4. $g^2(g^3 + 2g^2 - 3g + 4)$

5. $(d^2)^3$

6. $\frac{v^5}{v^2}$

VII. Squares and square roots.

A. Learn the perfect squares:

1. They are, from 1^2 to 15^2 , 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225

2. Learn means memorize.

B. To find square roots,

1. Factor any perfect squares out of the expression under the radical

2. Take those square roots, then leave the rest under the radical.

$$\text{Ex: } \sqrt{98x^3} = \sqrt{(49)(2)(x)(x)(x)} = 7x\sqrt{2x}$$

C. Practice

1. $\sqrt{32}$

2. $\sqrt{450}$

3. $\sqrt{48}$

VIII. Polynomials

A. Multiplying: FOIL binomials

$$\text{Ex: } (x+3)(x-2) = x^2 - 2x + 3x - 6 = x^2 + x - 6$$

$$(a + b)(c + d) = ac + ad + bc + bd$$

- Practice:**
1. $(a+5)(a-6)$
 2. $(x + 4)^2$
 3. $(b-7)(b+7)$

B. Factoring: Breaking a polynomial into its factors

1. Remove the GCF, if there is one

2. Look for a pattern

a. Difference of Two Squares (DOTS): $a^2 - b^2 = (a+b)(a-b)$

b. Perfect Trinomial Square (PTS): $a^2+2ab + b^2 = (a+b)(a+b) = (a+b)^2$

or $a^2-2ab + b^2 = (a-b)(a-b) = (a-b)^2$

$$\text{Ex: } x^2 - x - 6 = (x-3)(x+2)$$

$$\text{Ex: } 3x^3 - 3x^2 - 18x = 3x(x^2 - x - 6) = 3x(x-3)(x+2)$$

$$\text{Ex: } x^2 - 81 = (x+9)(x-9) \text{ (DOTS)}$$

$$\text{Ex: } x^2 + 6x + 9 = (x+3)(x+3) = (x+3)^2 \text{ (PTS)}$$

3. Practice: Factor completely

1. $4x^2 - 16$

2. $x^2 - 8x + 16$

3. $5x^2y - xy^2 + 3x^2y^2$

4. $b^2 - 38b + 72$

5. $15a^2 - 21a + 9$

6. $121x^2 - 81$

7. $4k^2 + 20k + 24$

8. $x^2 - 11x - 26$

IX. Solving quadratic equations (Equations like the ones in VIII)

A. Factor, then set each factor equal to 0 and solve the equations you get.

B. OR use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The original equation would be $ax^2 + bx + c = 0$

NOTE: a, b, and c are numbers, not variables.

C. The solutions to a quadratic equation are the x values that would make the graph of the equation cross the x axis, so if $ax^2 + bx + c = y$, they the solutions would be where $y = 0$. Some solutions are imaginary, but that is for later.

D. Practice

1. $x^2 - 2x - 8 = 0$ (Factor)
2. $2x^2 + 3x + 7 = 0$ (Use quadratic formula, leave in square-root form if you need to.)

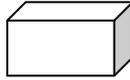
X. Geometry is based on logical reasoning from accepted principles and definitions.

A. Some definitions:

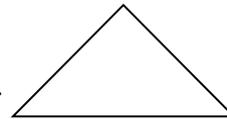
1. Point: has position only, no size; usually represented by a dot; always denoted with a capital letter.
2. Line: has position and length, designated by lower case letter or by points on it. Also represented by \leftrightarrow ; length is infinite
3. Plane: has position, length and width (both infinite), designated by three points or fancy (often italicized) capital letter.
4. Segment: a portion of a line, has a finite length.
5. Circle: the set of all points equidistant from a central point.
6. Triangle: a plane figure with three sides.
7. Quadrilateral: a plane figure with four sides (and all corners pointing out)

B. Some more general knowledge:

1. Two points determine a line; three points not in a line determine a plane.
2. Plane geometry deals with flat figures (i.e., figures on a plane) such as triangles, circles, squares, etc.
3. Solid geometry deals with three-dimensional figures such as cubes, spheres, pyramids, etc.



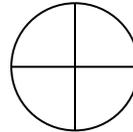
4. The angles of a triangle always add up to 180° .



5. A right angle has 90° .

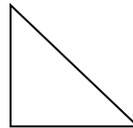


6. A circle has 360° .



7. Perimeter and circumference both mean the measure around a figure; circumference is used for circles.

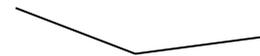
8. A right triangle has one right angle.



9. An acute ("sharp") angle measures less than 90° .



10. An obtuse ("dull") angle measures more than 90° .



XI. A few mixed review problems to **practice**, (**remember PEMDAS!**)

Simplify

1. $11(9) + 3(11)$

2. $(17-8) \div 3$

3. $42 \div 7 \div 2 + 1$

4. $-13+10$

5. $45-3+14-26$

6. $4+3-16+(-5)$

7. $245 + -15$

8. $-256-13+5$